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A SKETCHPAD INVESTIGATION OF 'ARITHMETIC SEQUENCE' PARABOLA

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Target Audience: GET and FET **Duration**: 2 hours
Required: Computer Lab with *Sketchpad* 4 plus data projector & OHP
Max. Participants: Dependent on no. of computers - max. 2 per computer

This workshop involves a *Sketchpad* investigation for secondary school learners adapted from Winicki-Landman (2001; 2002) that may provide a useful opportunity within an algebraic context for engaging them in some pattern recognition, generalising, conjecturing, refutation, and proof. During the activity learners may experience and understand some of the roles played by proof in mathematics such as verification, explanation, discovery and intellectual challenge.

The activity involves the investigation of some families of quadratic functions, discovering, formulating and proving some surprising properties related to them. Extensive use will be made of the function graphing facility of *Sketchpad* to provide dynamic visual stimuli to enhance and foster connections among different representations - symbolic, numeric and graphic.

Overall the activity is intended to elicit some mathematical surprises and feelings such as "*How come? Why? I don't believe it - WOW!*" Time permitting, participants might also be guided to some further generalisations involving translations and enlargements of parabola in general.

Reference

Winicki-Landman, G. (2001). Searching Families as a Source of Surprise. *Mathematics Teacher*, 94, 468-478.

Winicki-Landman, G. (2002). Families of functions and functions of proof. *Int. J. Math. Ed. Sci. & Technol.*, 33(6), 843-863.

WORKSHEETS

A Sketchpad Investigation of 'Arithmetic Sequence' Parabola

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1. Carefully examine the five quadratic equations in Family A (see attached sheet).

   (a) Can you suggest more functions that in your opinion belong to the same family?

   (b) Can you find a common relationship between the coefficients of this family of quadratic equations?

   (c) Can you conjecture a relationship among the graphs of this family of functions?

   (d) Check your conjecture by plotting all five functions on the same axis with *Sketchpad*. What do you notice?

   (e) Can you prove your conjecture?

2. Now examine the families B, C and D in the table.

   (a) What conjecture(s) can you now make for these families?

   (b) Check your conjecture(s) by plotting each family on the same axis with *Sketchpad*. What do you notice?

   (c) Can you prove your conjecture(s)?

   (d) Can you generalise and prove your generalisation?

3. Repeat Question 1 above for family E, and then Question 2 for families F, G and H.

4. Repeat Question 1 above for family I, and then Question 2 for families J, K and L.

Some Further Investigations

5. What are the loci of the turning points (vertices) of the graphs of three sets of families of quadratic functions above? (Use the loci facility on *Sketchpad* to investigate and determine their equations).

6. (a) Using *Sketchpad* graph a few examples of the family of quadratic functions of the form $y = x^2 + bx + 1 + b$. What do you notice? Can you prove your observation?

   (b) Repeat (a) above with families of functions of the form $y = 2x^2 + bx + 2 + b$ and $y = 3x^2 + bx + 3 + b$. Can you generalise and prove your observations?

7. (a) Choose an arbitrary quadratic function, say, $y = x^2 - 2x$, and graph it using *Sketchpad*.

   (b) Translate the parabola by 1 unit in the *x*-direction and -1 unit in the *y*-direction.

   (c) Translate the parabola by 2 units in the *x*-direction and -4 units in the *y*-direction.

   (d) Translate the parabola by 3 units in the *x*-direction and -9 units in the *y*-direction.

   (e) What do you notice? Can you prove your observation?

8. (a) Repeat Question 7 above for at least another two quadratic functions of the form $y = x^2 + bx + c$.

   (b) What do you notice?

   (c) Can you generalise and prove your observations?

9. Use the generalisation in Question 8 to provide an alternative proof of your observations in Question 1.

NOTE: A zipped *Sketchpad* document that contains sketches that can be used to explore the above investigations can be downloaded from http://mzone.mweb.co.za/residents/profmd/work.zip

Table: **FAMILIES OF FUNCTIONS**

FAMILY A	FAMILY B	FAMILY C	FAMILY D
$y = x^2 + x + 1$ $y = x^2 - 1$ $y = x^2 + 2x + 3$ $y = x^2 - x - 3$ $y = x^2 - 4x - 9$	$y = 2x^2 + 2x + 2$ $y = 2x^2 + 3x + 4$ $y = 2x^2 + x$ $y = 2x^2 - 2$ $y = 2x^2 + 5x + 8$	$y = -x^2 - x - 1$ $y = -x^2 + x + 3$ $y = -x^2 + 1$ $y = -x^2 - 2x - 3$ $y = -x^2 - 3x - 5$	$y = -4x^2 - 4x - 4$ $y = -4x^2 - 3x - 2$ $y = -4x^2 + 4$ $y = -4x^2 - 5x - 6$ $y = -4x^2 - 2x$
FAMILY E	**FAMILY F**	**FAMILY G**	**FAMILY H**
$y = x^2 + x + 1$ $y = 2x^2 + x$ $y = -x^2 + x + 3$ $y = 3x^2 + x - 1$ $y = 5x^2 + x - 3$	$y = 5x^2 + 5x + 5$ $y = 4x^2 + 5x + 6$ $y = 6x^2 + 5x + 4$ $y = 3x^2 + 5x + 7$ $y = -x^2 + 5x + 11$	$y = -2x^2 - 2x - 2$ $y = -4x^2 - 2x$ $y = -3x^2 - 2x - 1$ $y = 2x^2 - 2x - 6$ $y = 5x^2 - 2x - 9$	$y = x^2 - 1$ $y = -2x^2 + 2$ $y = 3x^2 - 3$ $y = -5x^2 + 5$ $y = -4x^2 + 4$
FAMILY I	**FAMILY J**	**FAMILY K**	**FAMILY L**
$y = x^2 + x + 1$ $y = 3x^2 + 2x + 1$ $y = -x^2 + 1$ $y = -3x^2 - x + 1$ $y = 5x^2 + 3x + 1$	$y = 2x^2 + 2x + 2$ $y = 6x^2 + 4x + 2$ $y = -2x^2 + 2$ $y = 4x^2 + 3x + 2$ $y = -4x^2 - x + 2$	$y = -3x^2 - 3x - 3$ $y = -5x^2 - 4x - 3$ $y = 3x^2 - 3$ $y = 9x^2 + 3x - 3$ $y = -11x^2 - 7x - 3$	$y = 4x^2 + 2x$ $y = 2x^2 + x$ $y = -5x^2 - 2.5x$ $y = 3x^2 + 1.5x$ $y = -6x^2 - 3$