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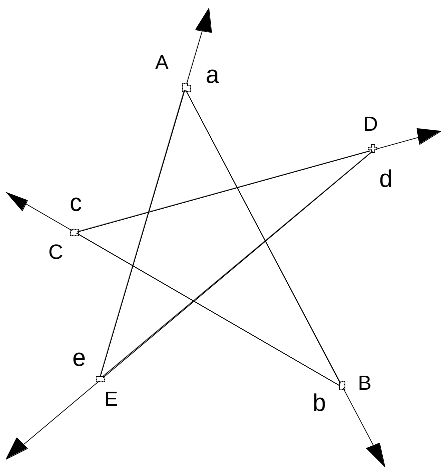
Stars: A Second Look

Star polygons as presented by Winicki-Landman (1999) certainly provide an excellent opportunity for students for investigating, conjecturing, refuting and explaining (proving). However, it could also be insightful to alternatively explain (prove) the results in terms of the exterior angles of the star polygons. It is also likely that students who have had a strong experiential background in LOGO would find this approach quite natural and easy. For example, consider the star pentagon shown below. Imagine that one is a turtle (like in turtle geometry) starting from A, then walking along the perimeter from A to B, turning through the exterior angle *b*, then from B to C, turning through exterior angle *c*, etc. When one returns to A, turning through the exterior angle *a*, one again faces in the same direction one started off from. The total turning undergone is two full revolutions (use a pen or pencil and consider the turn at the vertices), therefore: $a+b+c+d+e=720°$.



Since $a=180°-\angle A$, etc., the interior angle sum can now easily be determined as follows:

$$(180°-\angle A)+(180°-\angle B)+(180°-\angle C)+(180°-\angle D)+(180°-\angle E)=720°$$
$$\Leftrightarrow \angle A+\angle B+\angle C+\angle D+\angle E=5\bullet 180°-4\bullet 180°=180°$$

The value of this approach is that it is almost immediately generalizable to *any* closed polygon (of which star polygons are only a special case) as follows. In general, after walking around the perimeter of the polygon, one is facing in the same direction one has started off from, the total turning (sum of the turning angles) must be a multiple of 360º, ie. *k*.360º where *k* = 0; 1; 2; 3; etc. The sum of the interior angles is now simply

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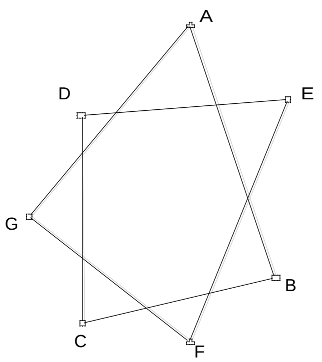
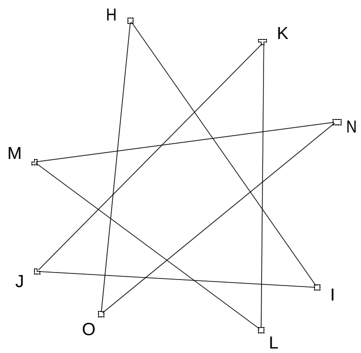
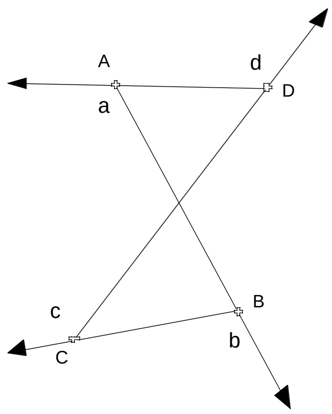
the difference between $n.180^\circ$ and the sum of the turning angles where n is the number of vertices, for example:

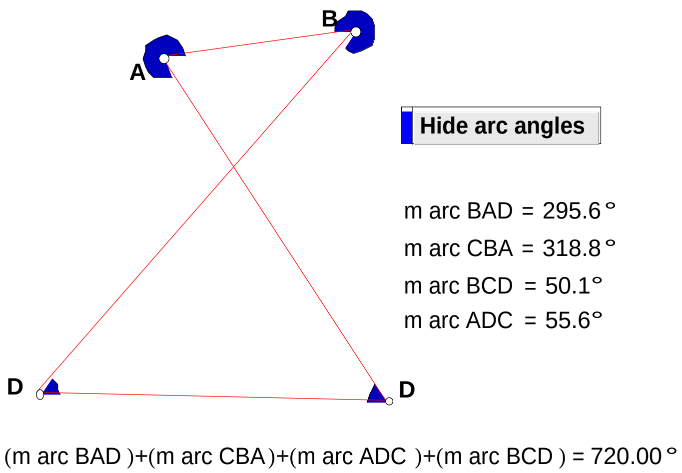
$$S = n \bullet 180^\circ - k \bullet 360^\circ$$
$$= 180^\circ(n - 2k)$$

Using this formula one can now easily determine the interior angle sums of the septagon and octagon shown below. In the first case, the value of k is 2 and in the second it is 3. The respective interior angle sums are therefore 540º and 360º. This formula, however, becomes even more useful if one is working with more complicated closed polygons, such as a crossed quadrilateral as shown below. In this case, the total turning $k = 0$, since one first undergoes two clockwise turns at B and C, but these are cancelled out by the two anti-clockwise turns at D and A respectively. The interior angle sum of the crossed quadrilateral is therefore 180º x 4 = 720º, and two of the "*interior*" angles now lie "*outside*"! This is usually very surprising to children and adults alike, and is a good example of the Lakatosian heuristic. (This is nicely illustrated in De Villiers, 1999 where arc measurements are used in *Sketchpad* for the crossed quadrilateral - see last figure below).









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