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Ezit's Rule: An Explanation and Proof

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At the recent AMESA Congress in Potchefstroom during July 2004, Connie Skelton from Maskew Miller Longman, told me about the following interesting discovery of a learner at Bridge House College in Cape Town, while she was still teaching mathematics there. Apparently the learners had been asked to explore some angle properties of a non-regular pentagonal star as shown in Figure 1.

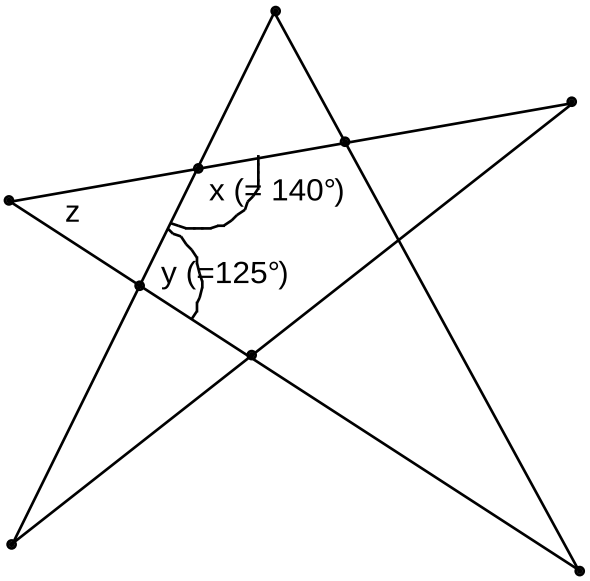

Figure 1

One obvious relationship she hoped learners would discover was that $\hat{z} = 140° + 125° - 180° = 265° - 180° = 85°$ (and eventually the generalization that $\hat{z} = \hat{x} + \hat{y} - 180°$). However, much to her surprise one learner, Ezit Malan, came up with the following unexpected rule that seemed to work perfectly well:

"*Add up the unit digits of the angles 140˚ and 125˚, 0 and 5, to get 5 as the unit digit for the required angle. To determine the tens digit of the required angle, add up the value of the other digits, 1, 4, 1 and 2, to give 8. This then gives the correct answer 85*."

Although Connie and her class did later find cases where Ezit's rule didn't work, it was still very puzzling that it seemed to work most of the time. This clearly left them with an unresolved feeling of curiosity, i.e. a feeling of "*When or why does it work?*"

I must also confess that when Connie first told me of Ezit's rule, I instinctively felt some disbelief. The strategy just seemed too arbitrary and inexplicable that my initial gut feeling was that it was only coincidental. However, after exploring her rule a while by considering a few different cases (and the reader is invited to do the same!), I quickly became convinced that there was certainly more to it than met the eye.

What now follows is an explanation and proof of when and why the result works, and that only requires elementary algebra and would be suitable for all learners at the junior secondary phase. However, let us first look at one or two more examples.

Some More Examples

Let us assume values of x and y between 0° and 180°. Note that for an angle z to be formed as shown, $x + y > 180^\circ$. It is further useful to distinguish the following three cases and to investigate each separately:

(1) both x and y are equal to or greater than 100°,

(2) one of x and y is smaller than 100°, while the other is greater than or equal to 100°,

(3) both x and y are smaller than 100°.

Case 1

Consider $x = 123^\circ$ and $y = 148^\circ$. Then $\hat{z} = 123^\circ + 148^\circ - 180^\circ = 91^\circ$.

According to Ezit's rule: Add the units 3 and 8 of the two angles to obtain 11 as the units of the desired angle. Next add the digits 1, 2, 1 and 4, to obtain 8, which together with 1 carried over from the 11 of the units, gives us the desired tens digits 9. So the angle is 91°.

It is left to the reader to check Ezit's Rule with a few more cases where both x and y are equal to or greater than 100°, and convince themselves that it is always true in this case.

Case 2

(a) Consider $x = 120^\circ$ and $y = 95^\circ$. Then $\hat{z} = 120^\circ + 95^\circ - 180^\circ = 35^\circ$.

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According to Ezit's rule: Add the units 0 and 5 of the two angles to obtain 5 as the units of the desired angle. Next add the digits 1, 2, and 9, to obtain 12, which gives us a tens digits of 12. However, an angle of 125˚ is wrong! So Ezit's rule is wrong in this case!

But wait! If we add the digits 1 and 2 of 12, we do get the desired value of 3 for the tens digits and obtain the correct angle 35˚! So maybe all we need to do is to continue adding the digits until we get a single digit answer in this case?

(b) Consider $x = 125^\circ$ and $y = 60^\circ$. Then $\hat{z} = 125^\circ + 60^\circ - 180^\circ = 5^\circ$.

According to Ezit's rule: Add the units 5 and 0 of the two angles to obtain 5 as the units of the desired angle. Next add the digits 1, 2, and 6, to obtain 9, which gives us an incorrect tens digits of 9. So here the rule clearly does not work (and no further adding of digits will help here)!

For case 2, we therefore seem to find two different scenarios: one for which Ezit's Rule works (if applied again), and another for which it doesn't. Under which conditions does the rule work (and under which does it not)? This question is for the moment left for the reader to investigate further.

Case 3

(a) Consider $x = 99^\circ$ and $y = 84^\circ$. Then $\hat{z} = 99^\circ + 84^\circ - 180^\circ = 3^\circ$.

According to Ezit's rule: Add the units 9 and 4 of the two angles to obtain 13 as the units of the desired angle. Next add the digits 9 and 8 to obtain 17, which together with 1 carried over from the 13 of the units, gives us a tens digits of 18. Again adding the digits 1 and 8, gives an incorrect tens digit of 9! So here the rule clearly also does not work !

(b) Consider $x = 95^\circ$ and $y = 97^\circ$. Then $\hat{z} = 95^\circ + 97^\circ - 180^\circ = 12^\circ$.

According to Ezit's rule: Add the units 5 and 7 of the two angles to obtain 12 as the units of the desired angle. Next add the digits 9 and 9 to obtain 18, which together with the 1 carried over from the 12, gives us an incorrect tens digits of 19. However, again adding the digits 1 and 9, gives 10, and adding the digits again, gives the required value of 1 for the tens digit, and the correct angle of 12˚. So here the rule (if applied until only one digit remains for the tens) also works!

For case 3, we therefore also seem to find two different scenarios: one for which Ezit's Rule works (if applied again), and another for which it doesn't. Under which conditions does the rule work (and under which does it not)? This question is for the moment also left for the reader to investigate further.

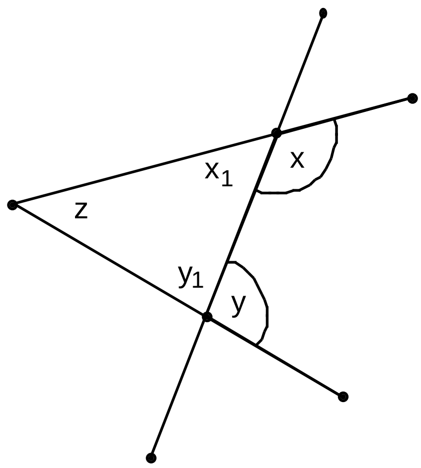

Figure 2

Explanation 1

For the sake of completeness, let us first explain (prove) why $\hat{z} = \hat{x} + \hat{y} - 180°$. With reference to Figure 2, we have from the triangle exterior angle theorem and the sum of the angles of a triangle:

$x = \hat{z} + \hat{y}_1$

$y = \hat{z} + \hat{x}_1$

$\therefore x + y = z + z + \hat{x}_1 + \hat{y}_1$

$\therefore x + y - (z + \hat{x}_1 + \hat{y}_1) = z$

$\therefore x + y - 180° = z.$

Explanation: Case 1

Consider digits *a* and *c*, and *b* and *d*, as the respective tens and units digits of *x* and *y*.

Thus $x = 100 + 10a + b$

$y = 100 + 10c + d$

Therefore $z = 200 + 10(a + c) + b + d - 180°$

$= 20° + 10(a + c) + b + d$

$= 10(a + c + 2) + b + d.$

According to Ezit's Rule, we must add the unit digits b and d of angles x and y, which gives $b + d$. Then we must add the hundreds and tens digits, 1, a, 1, and c to obtain the tens digit, which gives 10(a + c + 2). Therefore, the required angle z is $10(a+c+2)+b+d$, which is exactly the same as above. Q.E.D.

Explanation: Case 2

Let $x = 100 + 10a + b$

$y = 10c + d$

Therefore $z = 100 + 10(a + c) + b + d - 180°$

$= -80° + 10(a + c) + b + d$

$= 10(a + c - 8) + b + d.$

According to Ezit's Rule, we must add the unit digits b and d of angles x and y, which gives $b + d$. Then we must add the hundreds and tens digits, 1, a, and c, which gives 10(a + c + 1). Therefore, the angle z is $10(a+c+1)+b+d$, but that is clearly not equivalent to the correct angle shown above. This therefore shows why in this case Ezit's rule does not always work.

But when does Ezit's Rule still work? If we rewrite the correct value of z as $10(a+c+1-9)+b+d$, the relationship between the two formulae becomes clearer. Indeed, they will be equivalent under the condition that $a + c + 1 > 9$ or more simply $a + c > 8$. Probably the most straightforward way to verify this without using modular arithmetic, is simply by quickly checking all cases for the values of the tens digits. Since both a and c are equal to or smaller than 9, means we only need to check values of $a + c$ from 9 to 18. For example, consider the following table (with the rest of the table left for the reader to complete).

a + c	Correct Value (a + c -8)	Ezit's Rule (a + c + 1)	Applied again (Sum of digits)
9	1	10	1
10	2	11	2
11	3	12	3
12	4	13	4
13			
14			

15			
16			
17			
18	10	19	10

Explanation: Case 3

As before we can use algebra to show that in this case, Ezit's Rule is not generally true (and is left to the reader). Furthermore, in this case where both x and y are smaller than 100˚, Ezit's Rule will only work under the condition that a and c are both 9, and $b + d > 9$. This can be verified easily by again checking all cases.

Alternatively, note that for Ezit's Rule to work in this case, a and c need to add up to 19 to obtain a 10, and eventually a 1, when adding up the digits. This is obviously only possible if a and c are both 9, and $b + d > 9$.

Concluding Comments

The preceding analysis gives a good example of the type of dynamic interplay between experimentation and deductive thinking that regularly happens in mathematical research (compare De Villiers, 2003, 2004), but is often sadly hidden from our learners. Traditional talk-and-chalk assumes that learners cannot rediscover or invent any mathematics on their own, and therefore all content is presented to learners ready-made and neatly packaged. It is therefore most welcome that the new OBE curriculum stresses that learners should be given opportunities to engage in some more open-ended investigations of the kind discussed here.

As a whole, this episode shows that our learners are quite capable of making some original discoveries of their own, provided they are given the opportunity. Such a classroom discovery and discussion, rare as it may be, can positively contribute to demystifying mathematics as a static, stale subject, and bring learners closer to the true nature of mathematical thinking. Moreover, such episodes would allow learners to develop a sense of ownership over results that is not only motivating, but also empowering them into believing in their own ability to create mathematics, rather than just memorising given rules and procedures. After all, mathematics is far more about the process of doing mathematics (e.g. experimenting, conjecturing, investigating, refuting, explaining, etc.) than about the cut-and-dried end-products which are examined at the end of the year.

References

De Villiers, M. (2003). The value of experimentation in mathematics. *AMESA Proceedings*, July 2003, Cape Town, pp. 174-185. (A 40 KB pdf version of this paper can be directly downloaded from: http://mysite.mweb.co.za/residents/profmd/experiment.pdf)

De Villiers, M. (2004). The role and function of quasi-empirical methods in mathematics. *Canadian Journal of Science, Mathematics & Technology Education*, Vol. 4, No. 3, July, pp. 397-418. (A 316 KB pdf version of this paper can be directly downloaded from: http://mysite.mweb.co.za/residents/profmd/experiment2.pdf)